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Wages, Employment, and the Production Function

*Hans Brems*

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FACULTY WORKING PAPER NO. 1477


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August 1988

Wages, Employment, and the Production Function

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## WAGES, EMPLOYMENT, AND THE PRODUCTION FUNCTION

HANS BREMS

### Abstract

Monetarist and New Classical writers rarely mention their production function, let alone its functional form. The present paper specifies the empirically robust Cobb-Douglas form, derives a very elastic demand for labor from it, and sees the result within the framework of a Friedman "natural" rate of unemployment. The model is solved for its employment, real wage rate, physical output, distributive shares, and its nominal income, wage rate, and price. The algebra is remarkably simple: all elasticities of solutions with respect to policy instruments are constant.



## 1. Introduction

Consider a short-run static model of a private and closed economy. Macroeconomic theory, in effect, imagines such an economy producing a single good: physical output and its price are then well-defined and tractable variables. But even with a single good we must be careful with our aggregation. An aggregate demand for labor must be explicitly derived from firm demand for labor. Firm demand for labor must be explicitly derived from the production function faced by the firm, and so must an aggregate production function. We derive all of them from the empirically robust Cobb-Douglas production function, find the demand for labor to be very elastic with respect to the real wage rate, and see our results in a Friedman framework of a "natural" rate. We solve the resulting model for its employment, real wage rate, physical output, distributive shares, and its nominal income, wage rate, and price. To our delight the sensitivities of such solutions to the policy instruments of the model may be measured by constant elasticities.

We use the following variables and parameters.

2. Variables

$\kappa$   $\equiv$  physical marginal productivity of capital stock

$L$   $\equiv$  labor employed

$P$   $\equiv$  price

$W$   $\equiv$  money wage bill

$w$   $\equiv$  money wage rate

$X$   $\equiv$  physical output

$Y$   $\equiv$  money national income

$Z$   $\equiv$  money profits bill

3. Parameters

$a$   $\equiv$  joint factor productivity

$\alpha$   $\equiv$  elasticity of physical output with respect to labor

$\beta$   $\equiv$  elasticity of physical output with respect to capital stock

$F$   $\equiv$  available labor force

$\lambda$   $\equiv$  fraction of available labor force employed

$M$   $\equiv$  supply of money

$S$   $\equiv$  physical capital stock

$V$   $\equiv$  velocity of money

#### 4. National Income and Output

Money national income defined as the aggregate earnings arising from current production is identically equal to national product defined as the market value of physical output:

$$Y \equiv PX \tag{1}$$

#### 5. Demand for Labor

We must be careful with our aggregation and begin at the firm level. Let the inputs of the  $i$ th firm be  $L_i$  and  $S_i$  and its output be  $X_i$ . Then under a Cobb-Douglas production function common to all firms the output of the  $i$ th firm will be

$$X_i = aL_i^\alpha S_i^\beta \tag{2}$$

where  $0 < \alpha < 1$ ,  $0 < \beta < 1$ ,  $\alpha + \beta = 1$ , and  $a$  is what growth measurement [Maddison (1987: 658)] calls "joint factor productivity."

Maximizing its profits the  $i$ th firm will hire labor until the last man costs as much as he contributes. Under pure competition, then,

the real wage rate will equal the physical marginal productivity of labor:

$$\frac{w}{P} = \frac{\partial X_i}{\partial L_i} = \alpha \alpha L_i^{\alpha-1} S_i^\beta = \alpha \frac{X_i}{L_i} \quad (3)$$

Here, as assumed,  $\alpha - 1 = -\beta$ , so raise both sides of (3) to the power  $-1/\beta$ , rearrange, and write firm demand for labor as an explicit function of the real wage rate:

$$L_i = (\alpha \alpha)^{1/\beta} \left(\frac{w}{P}\right)^{-1/\beta} S_i \quad (4)$$

Write (4) for the  $j$ th firm, divide the result by (4) for the  $i$ th firm, and find

$$L_j/L_i = S_j/S_i \equiv s_j$$

$$L_j = L_i s_j$$

$$S_j = S_i s_j$$

where  $s_j$  is a positive proper or improper fraction measuring the scale of the  $j$ th firm relative to that of the  $i$ th firm. The Cobb-Douglas production function (2), common to all firms, may then be written out for each of them,  $j = 1 \dots n$ , recalling that  $\alpha + \beta = 1$ :

$$\begin{aligned} X_1 &= aL_1^\alpha S_1^\beta = a(L_i s_1)^\alpha (S_i s_1)^\beta = aL_i^\alpha S_i^\beta s_1 \\ &\cdot \\ &\cdot \\ &\cdot \\ X_n &= aL_n^\alpha S_n^\beta = a(L_i s_n)^\alpha (S_i s_n)^\beta = aL_i^\alpha S_i^\beta s_n \end{aligned}$$

Add the  $n$  outputs,  $j = 1 \dots n$ :

$$\begin{aligned} X_1 + \dots + X_n &= aL_i^\alpha S_i^\beta (s_1 + \dots + s_n) \\ &= a[L_i(s_1 + \dots + s_n)]^\alpha [S_i(s_1 + \dots + s_n)]^\beta \\ &= a(L_1 + \dots + L_n)^\alpha (S_1 + \dots + S_n)^\beta \end{aligned}$$

Defining aggregate employment, capital stock, and output:

$$L_1 + \dots + L_n \equiv L$$

$$S_1 + \dots + S_n \equiv S$$

$$X_1 + \dots + X_n \equiv X$$

we aggregate our Cobb-Douglas production function (2) for  $i = 1 \dots n$  into

$$X = aL^\alpha S^\beta \tag{5}$$

and our firm demand for labor function (4) for  $i = 1 \dots n$  into

$$L = (a\alpha)^{1/\beta} \left(\frac{w}{P}\right)^{-1/\beta} S \tag{6}$$

whose real-wage-rate elasticity is seen to be:

$$\frac{\partial \log_e L}{\partial \log_e (w/P)} = -\frac{1}{\beta} \tag{7}$$



A realistic value of  $\beta$  would be  $1/4$ , so the real-wage-rate elasticity of the demand for labor would be  $-4$ . As shown in our appendix, contained in the logarithmic definition of the elasticity (7) is the better known definition of it as the percentage change in a dependent variable  $L$  for a given percentage change in an independent variable  $w/P$ . In plainer words, then, reducing the real wage rate  $w/P$  by one percent would raise aggregate demand for labor by four percent.

Inherent in our empirically robust Cobb-Douglas function (5) is a unitary elasticity of substitution between labor and capital. Demand for good substitutes can be expected to be elastic--as we have found.

## 6. Solution for Employment

Current labor-market literature, e.g., Lindbeck and Snower (1986) and Blanchard and Summers (1988) distinguish between "insiders," who are employed hence decision-making, and "outsiders," who are unemployed hence disenfranchised. Facing our demand (6) the decision-making insiders can have a higher real wage rate for themselves by accepting less employment of the disenfranchised outsiders. The resulting unemployment is involuntary to the outsiders but voluntary to the insiders, who accept it with the better conscience the more generous the unemployment insurance benefits [Casson (1984)]--and the more unemployment is perceived to be Keynesian!

TABLE I. UNION DENSITY

Union Proportion of Nonagricultural Employees, 1984-1985

Low	Medium	High
0-30	30-45	45-100
U.S.	Switzerland	Italy
France	Netherlands	U.K.
Japan	Germany	Sweden

*Source:* Freeman (1988: 69).

TABLE II. MILITANCY

Working Days Lost Per 1,000 Employees, Annual Average 1977-1986

Low	Medium	High
0-50	50-175	175-1,000
Switzerland	Germany	U.S.
Japan	France	U.K.
Netherlands	Sweden	Italy

*Source:* I.L.O.

TABLE III. UNEMPLOYMENT

Unemployment Rates, National Definitions, 1986

Low	Medium	High
0-4	4-11	11-14
Switzerland	U.S.	Italy
Sweden	Germany	U.K.
Japan	France	Netherlands

*Source:* O.E.C.D.

Everything taken into account, let insiders accept the fraction  $\lambda$  employed of available labor force, where  $0 < \lambda \leq 1$ . In other words, if  $L > \lambda F$  insiders will insist on a higher real wage rate. If

$$L = \lambda F \tag{8}$$

they will be happy with the existing real wage rate. If  $L < \lambda F$  they will settle for a lower real wage rate.

Consider the fraction  $\lambda$  a parameter, then (8) will be a solution for employment corresponding to Friedman's (1968: 8) "natural" rate of unemployment. The fraction  $\lambda$  must reflect institutional dimensions of the labor market. In a nonunionized labor market, for example, the real wage rate would simply be an equilibrating variable clearing the labor market, hence the natural rate of employment would be at its upper limit  $\lambda = 1$ . It would be the lower, i.e., unemployment  $1 - \lambda$  be the higher, the more unionized the labor market is and the more militant its unions are. Or would it?

Tables I-III classify nine advanced economies as "low," "medium," or "high" in each of three dimensions of their labor markets: union density, militancy, and unemployment. The brackets of each dimension were defined such that groups of three economies would occupy each bracket. Within each bracket the group was arranged in ascending order of the dimension in question.

Four of the nine economies display full correlation between union density, militancy, and unemployment in the sense that the economy occupies the same bracket in all three dimensions: Japan always the lower bracket, Germany always the medium bracket, and Italy and the U.K. always the higher bracket.

Six of the nine economies display correlation between militancy and unemployment in the sense that the economy occupies the same bracket in these two dimensions: Japan and Switzerland the lower bracket, France and Germany the medium bracket, and Italy and the U.K. the higher bracket.

Three economies display no correlation at all: the Netherlands, Sweden, and the U.S. all manage to occupy at the same time three different brackets in the three dimensions.

#### 7. Solution for the Real Wage Rate

Raise (6) to the power  $-\beta$ , insert (8), rearrange, and solve for the real wage rate

$$\frac{w}{p} = a\alpha(\lambda F)^{-\beta} S^{\beta} \quad (9)$$

What is the implied slope of the Phillips curve? The solution (9) was found by inserting (8), hence is the real wage rate insiders will be happy with, given their natural rate of employment  $\lambda$ . As long as the ratio  $w/P$  satisfies (9) the levels of the money wage rate  $w$  and price  $P$  can be anything: the Phillips curve is vertical.

But can labor negotiate real rather than money wages? Under indexation it can, and contract periods may be long. Where indexation is illegal, not practiced, or practiced incompletely, shorter contract periods will have to do. Then a temporarily finite slope of the Phillips curve is possible. For example, let the monetary authorities expand the money supply, thus stimulating demand, and let price respond more readily than does the money wage rate. Then firms will experience a temporary reduction of the real wage rate, temporarily reducing the actual rate of unemployment below its natural rate. But at the next round of collective bargaining, labor will restore the original real wage rate (9) and with it, a Friedman natural rate of unemployment  $1 - \lambda$ . Repeated attempts by government to apply Keynesian instruments to non-Keynesian unemployment will fail: instead of a long-run reduction of unemployment, they merely generate successive rounds of inflation.

8. Solution for Physical Output

Insert (8) into (5) and solve for physical output

$$X = a(\lambda F)^{\alpha} S^{\beta} \quad (10)$$

9. Solutions for Distributive Shares

Once again we must be careful with our aggregation and begin at the firm level. In (3) multiply by  $PL_i$  and define the wage bill of the  $i$ th firm

$$W_i \equiv wL_i = \alpha PX_i$$

Summing over  $n$  firms define

$$\sum_{i=1}^n W_i \equiv W$$

and find aggregate wage bill

$$W = \alpha PX$$

Finally use (1) to write labor's share

$$\frac{W}{Y} = \alpha \quad (11)$$

Define physical marginal productivity of capital stock

$$\kappa \equiv \frac{\partial X_i}{\partial S_i} = a\beta L_i^\alpha S_i^{\beta-1} = \beta \frac{X_i}{S_i}$$

Multiply by  $PS_i$  and define the profits bill of the  $i$ th firm

$$Z_i \equiv \kappa PS_i = \beta PX_i$$

Summing over  $n$  firms define

$$\sum_{i=1}^n Z_i \equiv Z$$

and find aggregate profits bill

$$Z = \beta PX$$

Finally use (1) to write capital's share

$$\frac{Z}{Y} = \beta \quad (12)$$

10. Solutions for Levels of Nominal Variables

So far, like Walras, we have written a system (1) through (12) which is homogeneous of degree zero in its nominal variables, hence cannot determine their levels  $P$ ,  $w$ , and  $Y$ . To determine such levels we must, first, define the velocity of money as the number of times per year a stock of money transacts money national income:

$$Y \equiv MV \quad (13)$$

and, second, consider the money supply  $M$  and the velocity  $V$  to be parameters. In that case (13) has trapped the level of money national income  $Y$  and is a solution for it.

Insert (8) and (13) into (11) and solve for the level of the money wage rate



$$w = \alpha \frac{MV}{\lambda F} \quad (14)$$

Insert (1) and (10) into (13) and solve for the level of price

$$P = \frac{MV}{a(\lambda F)^\alpha S^\beta} \quad (15)$$

## 11. Elasticities

Let us examine the sensitivities of our solutions (8) through (15) to the two policy instruments  $\lambda$  and  $M$ .

We couldn't wish for a simpler measure of such sensitivities than the constant elasticities displayed by the log-linear form of our solutions. An elasticity is defined as the partial derivative of the natural logarithm of a solution with respect to the natural logarithm of a parameter. So we take natural logarithms of all solutions and then differentiate them partially with respect to  $\log_e \lambda$  and  $\log_e M$ :

$$\frac{\partial \log_e L}{\partial \log_e \lambda} = 1 \quad (16)$$

$$\frac{\partial \log_e L}{\partial \log_e M} = 0 \quad (17)$$

$$\frac{\partial \log_e (w/P)}{\partial \log_e \lambda} = -\beta \quad (18)$$

$$\frac{\partial \log_e (w/P)}{\partial \log_e M} = 0 \quad (19)$$

$$\frac{\partial \log_e X}{\partial \log_e \lambda} = \alpha \quad (20)$$

$$\frac{\partial \log_e X}{\partial \log_e M} = 0 \quad (21)$$

$$\frac{\partial \log_e (W/Y)}{\partial \log_e \lambda} = 0 \quad (22)$$

$$\frac{\partial \log_e (W/Y)}{\partial \log_e M} = 0 \quad (23)$$

$$\frac{\partial \log_e (Z/Y)}{\partial \log_e \lambda} = 0 \quad (24)$$

$$\frac{\partial \log_e (Z/Y)}{\partial \log_e M} = 0 \quad (25)$$

$$\frac{\partial \log_e Y}{\partial \log_e \lambda} = 0 \quad (26)$$

$$\frac{\partial \log_e Y}{\partial \log_e M} = 1 \quad (27)$$

$$\frac{\partial \log_e w}{\partial \log_e \lambda} = -1 \quad (28)$$

$$\frac{\partial \log_e w}{\partial \log_e M} = 1 \quad (29)$$

$$\frac{\partial \log_e P}{\partial \log_e \lambda} = -\alpha \quad (30)$$

$$\frac{\partial \log_e P}{\partial \log_e M} = 1 \quad (31)$$

12. Summary of Findings

Of our sixteen elasticities half are zero, but zeros are also interesting. Across countries and over time, growth measurement [Denison (1967: 42) and (1974: 260)] found distributive shares remarkably constant. In good accordance our elasticities (22) through (25) are zero: our distributive shares are affected by neither the natural rate of employment nor the supply of money. In good accordance with monetarist doctrine [Friedman (1968)] our elasticities (17), (19), and (21) are zero: real variables are not affected by the money supply.

As for our nonzero elasticities we use  $\alpha = 3/4$  and  $\beta = 1/4$  and summarize our findings as follows.

Reducing the natural rate of employment  $\lambda$  by one percent would, in (16), reduce employment  $L$  by one percent; in (18) raise the real wage rate  $w/P$  by  $1/4$  percent; in (20) reduce physical output  $X$  by  $3/4$  percent; in (28) raise the money wage rate  $w$  by one percent; and in (30) raise price  $P$  by  $3/4$  percent.

Expanding the money supply  $M$  by one percent would, in (27), (29), and (31) raise money national income  $Y$ , the money wage rate  $w$ , and price  $P$  by one percent--as Hume [1752 (1875: 333)] said it would!

13. The Long Run

Our static elasticity (7) was a partial derivative, meaning that everything else in (6) was being kept constant such as the joint factor productivity  $a$ , the elasticities  $\alpha$  and  $\beta$ , and the physical capital stock  $S$ . Our short-run static model indeed kept them constant. In long-run U.S. growth they have not remained constant. Can our static elasticity (7) find its place in such a growth context, and is it consistent with stylized facts?

Define the proportionate rate of growth of a variable  $v$  as the derivative of its logarithm with respect to time:

$$g_v \equiv \frac{d \log_e v}{dt} \tag{32}$$

Then take the logarithm of (6). Consider the joint factor productivity  $a$ , the real wage rate  $w/P$ , and physical capital stock  $S$  functions of time but consider the elasticities  $\alpha$  and  $\beta$  stationary, i.e., having zero growth rates. Differentiate the logarithm of (6) with respect to time, use (32), and find

$$g_L = \frac{1}{\beta} g_a - \frac{1}{\beta} g_{w/P} + g_S \quad (33)$$

Stylized facts of long-run U.S. growth are

$$g_a = 0.015$$

$$g_{w/P} = 0.02$$

$$g_S = 0.03$$

Into (33) insert these stylized facts along with our  $\beta = 1/4$  and find  $g_L = 0.01$ --another stylized fact of long-run U.S. growth. Thus our static elasticity (7) has found its place in a growth context and is consistent with stylized facts.

All a crude two-dimensional time-series approach would see was that for a century the real wage rate was growing twice as rapidly as employment--a positive correlation between  $g_{w/P}$  and  $g_L$ ! More sophisticated multi-dimensional time-series approaches such as those by Yndgaard (1982) for one economy, Symons-Layard (1984) for six economies, and Klodt (1986) for 19 economies saw nothing but negative wage elasticities of employment--but none quite as negative as our uncompromising static elasticity (7).

14. The Rate of Interest

We have managed to do without a rate of interest, without a propensity to consume or save, and without a distinction between consumption and investment. The underlying tacit assumption was a well-functioning capital market in which a flexible rate of interest would be the equilibrating variable between saving and investment. In that case the propensities to consume or save would be of no direct consequence for the aggregate demand for our single good. A higher propensity to save would simply reduce one use, i.e., consumption, of the single good but expand another, i.e., investment, by as much--as Smith<sup>1</sup> [1776 (1805: 78-79)] and Ricardo<sup>2</sup> (1951, IV: 179-180) said it would. The New Classical economics is indeed classical!

In the long run there would be important indirect consequences for physical product in the form of capital widening, deepening, or quickening--beyond reach of our short-run model.

15. Disaggregation

We have cultivated the abstraction of an economy producing a single good sold at price P by firms applying a common production function and employing a single kind of labor at the common money wage

rate  $w$ . The resulting macroeconomic labor market, we found, might not clear.

Turning to microeconomics we find a reality displaying all sorts of wage differentials: wage differentials between industries, between firms in the same industry, between skilled and unskilled labor, between male and female labor. Unions in Europe and minimum-wage legislation in the United States try to narrow such wage differentials.

As a result, microeconomic labor markets may not clear either. As economists know, in the short run such narrowing will generate excess demand for labor priced too low. The symptom is wage drift. The narrowing will generate excess supply of labor priced too high. The symptom is chronic unemployment of, say, the dropouts or the elderly. In the long run, wage differentials too narrow to reflect differences in skill and education will reduce the incentives to acquire such skill and education.

FOOTNOTES

<sup>1</sup>"Whatever a person saves from his revenue he adds to his capital, and either employs it himself in maintaining an additional number of productive hands, or enables some other person to do so, by lending it to him for an interest...

"What is annually saved is as regularly consumed as what is annually spent, and nearly in the same time too; but it is consumed by a different set of people."

<sup>2</sup>"There is ... no danger that ... accumulated capital ... would not find employment. ... There are always to be found in a great country, a sufficient number of responsible persons, with the requisite skill, ready to employ the accumulated capital of others, and to pay them a share of the profits, and which, in all countries, is known by the name of interest for borrowed money."



APPENDIX. THE DEFINITION OF ELASTICITY

Contained in the logarithmic definition of an elasticity as  $d\log_e y/d\log_e x$  is the better known definition of it as  $(dy/dx)(x/y)$  or the percentage change in a dependent variable  $y$  for a given percentage change in an independent variable  $x$ . That one is contained in the other is easily shown.

Let all logarithms be natural ones. Let  $x \equiv e^u$  and  $y \equiv e^v$ . Then  $\log_e x \equiv u$  and  $\log_e y \equiv v$ . Use the chain rule

$$\frac{d\log_e y}{d\log_e x} \equiv \frac{dv}{du} \equiv \frac{dv}{dy} \frac{dy}{dx} \frac{dx}{du}$$

But  $dy/dv \equiv de^v/dv \equiv e^v \equiv y$ , so its reciprocal  $dv/dy \equiv 1/y$ . And  $dx/du \equiv de^u/du \equiv e^u \equiv x$ . Insert and find

$$\frac{d\log_e y}{d\log_e x} \equiv \frac{dy}{dx} \frac{x}{y},$$

so one definition is contained in the other.

REFERENCES

- Bain, George Sayers, and Price, Robert, Profiles of Union Growth--A Comparative Statistical Portrait of Eight Countries, Oxford: Blackwell, 1980.
- Blanchard, Oliver J., and Summers, Lawrence H., "Hysteresis and the European Unemployment Problem," in Cross, Rod (ed.) Unemployment, Hysteresis and the Natural Rate Hypothesis, Oxford: Blackwell, 1988.
- Casson, Mark, Economics of Unemployment, Cambridge, Mass.: MIT Press, 1984.
- Denison, Edward Fulton, Why Growth Rates Differ: Postwar Experience in Nine Western Countries, Washington, D.C.: Brookings Institution, 1967.
- \_\_\_\_\_, Accounting for United States Economic Growth 1929-1969, Washington, D.C.: Brookings Institution, 1974.
- Freeman, Richard B., "Contraction and Expansion: The Divergence of Private Sector and Public Sector Unionism in the United States," Journal of Economic Perspectives, Spring 1988, 2, 63-88.

Friedman, Milton, "The Role of Monetary Policy," Amer. Econ. Rev.,  
March 1968, 58, 1-17.

Hume, David, Political Discourses, Edinburgh: A. Kincaid & A.  
Donaldson, 1752, reprinted in Essays--Moral, Political, and  
Literary, vol. I, London: Longmans, Green, 1875.

International Labour Office, "Industrial Disputes, General Level,"  
I. L. O. Year Book of Labour Statistics 1987, Geneva: I. L. O.,  
1987, 963-975.

Klodt, Henning, "Lohnquote und Beschäftigung--Die Lohnlücke,"  
Jahrbücher für Nationalökonomie und Statistik, Sep. 1986, 201,  
480-497.

Lindbeck, Assar, and Snower, Dennis J., "Wage Setting, Unemployment,  
and Insider-Outsider Relations," Amer. Econ. Rev., May 1986, 76,  
235-239.

Maddison, Angus, "Growth and Slowdown in Advanced Capitalist Econo-  
mies: Techniques of Quantitative Assessment," J. Econ. Lit., June  
1987, 25, 649-698.

O. E. C. D., Labour Force Statistics, Paris: O. E. C. D., 1987.

\_\_\_\_\_, "Highlights," The O. E. C. D. Observer, January 1987,  
33-37.

Ricardo, David, Pamphlets and Papers, 1815-1823 reprinted in The Works  
and Correspondence of David Ricardo, P. Sraffa and M. H. Dobb  
(eds.), vol. IV, New York: Cambridge U. Press, 1951.

Smith, Adam, An Inquiry into the Nature and Causes of the Wealth of  
Nations, London: W. Strahan, 1776, "new" edition, Glasgow: R.  
Chapman, 1805.

Symons, J., and Layard, Richard, "Neoclassical Demand for Labour  
Functions for Six Major Economies," Econ. J., Dec. 1984, 94,  
788-799.

Windmuller, John P., "Comparative Study of Methods and Practices,"  
Collective Bargaining in Industrialised Market Economies, Geneva:  
I. L. O., 1987, 3-158, especially p. 19.

Yndgaard, Ebbe, "Danish Industry 1964-1979," Economic Essays in Honour of Jørgen H. Gelting, Copenhagen: Danish Economic Association, 1982, 192-206.



WAGES, EMPLOYMENT, AND THE PRODUCTION FUNCTION: NONPURE COMPETITION

HANS BREMS

Let all firms be producing the same good and selling it at the common price  $P$ . But let the  $i$ th firm be large enough to affect that price:

$$P = mX_i^\eta$$

whose constant elasticity  $0 > \eta > -1$  is the reciprocal of the price elasticity of demand and is common to all firms. Define gross profits as

$$Z_i \equiv PX_i - wL_i$$

and maximize it with respect to employment  $L_i$ :

$$\frac{\partial Z_i}{\partial L_i} = P \frac{\partial X_i}{\partial L_i} + X_i \frac{\partial P}{\partial X_i} \frac{\partial X_i}{\partial L_i} - w = (1 + \eta)P \frac{\partial X_i}{\partial L_i} - w = 0;$$

$$\frac{w}{P} = (1 + \eta) \frac{\partial X_i}{\partial L_i}$$

to be used instead of (3) on Page 5. Differentiate, raise to power  $-1/\beta$ , rearrange, aggregate over firms, and find aggregate demand for labor

$$L = [\alpha(1 + \eta)]^{1/\beta} \left(\frac{w}{P}\right)^{-1/\beta} S$$

to be used instead of (6) on Page 7. The real-wage-rate elasticity of aggregate demand for labor is still  $-1/\beta$ .









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